Algebra III

Name ______
Per ____ Date ______

Applications – Linear Equations

Business and Economics

In Exercises 58 and 59, assume that the data can be approximated by a straight line.

58. **Sales** The sales of a small company were $27,000 in its second year of operation and $63,000 in its fifth year. Let y represent sales in the xth year of operation.
   a. Find the slope of the sales line, and give an equation for the line in the form $y = mx + b$.
   b. Use your answer from part a to find out how many years must pass before the sales surpass $100,000.

59. **Manufacturing Costs** A company finds that it can make a total of 20 solar heaters for $13,900, while 10 solar heaters cost $7500 to make. Let y be the total cost to produce x solar heaters.
   a. Find the slope of the cost line, and give an equation for the line in the form $y = mx + b$.
   b. Use your answer from part a to find out how many solar heaters the company can make for $23,500.
   c. Use your answer from part a to find out how many solar heaters the company can make for $1000.

Life Sciences

60. **Effects of Pollution** When a certain industrial pollutant is dumped into a river, the reproduction of catfish declines. In a given period of time, dumping 3 tons of the pollutant results in a fish population of 37,000. Also, 12 tons of pollutant produce a fish population of 28,000. Let y be the fish population when x tons of pollutant are dumped into the river.
   a. Assuming there is a linear relationship between the amount of pollution in the river and catfish reproduction, write an equation of the line based on the given data.
   b. Predict the fish population if 15 tons of pollutant are dumped into the river.
   c. How many tons of the pollutant would result in elimination of the fish population?
   d. How much pollution should be allowed to maintain a fish population of 45,000?

61. **HIV Infection** The time interval between a person's initial infection with HIV and that person's eventual development of AIDS symptoms is an important issue. The method of infection with HIV affects the time interval before AIDS develops. One study of HIV patients who were infected by intravenous drug use found that 17% of the patients had AIDS after 4 years, and 33% had developed the disease after 7 years. The relationship between the time interval and the percent of patients with AIDS can be modeled accurately with a linear equation.*
   a. Write a linear equation $y = mx + b$ that models this data, using the ordered pairs (4, .17) and (7, .33).
   b. Use your equation from part a to predict the number of years before half of these patients will have AIDS.

62. **Estimating Height** A person's tibia bone goes from ankle to knee. A male with a tibia 40 cm in length will have a height of 177 cm, while a tibia 43 cm in length corresponds to a height of 185 cm.
   a. Write a linear equation showing how the height h of a male relates to the length t of his tibia.
   b. Suppose an archaeologist finds a collection of tibias having lengths of 38 to 45 cm. Assuming these belonged to males, estimate what their heights would have been.
   c. Estimate the length of the tibia for a height of 190 cm.

Social Sciences

63. **Voting Analysis** According to research by political scientist James March, if the Republicans win 45% of the two-party vote, they win 32.5% of the seats. If they win 60% of the vote, they get 70% of the seats. Let y represent the percent of the seats and x the percent of the vote.
   a. Write a linear equation satisfying this data.
   b. Use your equation from part a to predict the percent of Republican seats won if the Republicans get 50% of the vote.
   c. Find the percent of the vote the Republicans would need to capture 60% of the seats in the House of Representatives.
   d. Do you believe the equation found in part a is valid for every percent of the vote between 0% and 100%? Explain.

64. **Immigration** In 1974, 86,821 people from other countries immigrated to the state of California. In 1984, the number of immigrants was 140,289.
   a. If the change in foreign immigration to California is considered to be linear, write an equation expressing the number of immigrants, y, in terms of the number of years after 1974, x.
   b. Use your result in part a to predict the foreign immigration to California in the year 2001.

65. **Work-Related Illness and Injury** In 1943, the number of officially reported work-related illnesses and injuries in the state of California was 152,000, and in 1982 it was 330,870.
   a. If the change in the number of work-related illnesses and injuries is thought to be linear, write an equation expressing the number of illnesses and injuries, y, in terms of the number of years after 1943, x.
   b. Use your result in part a to predict the number of work-related illnesses and injuries in California in the year 2000.

58 (2, 27000) (5, 63000)

a) $\frac{63000-27000}{5-2} = \frac{36000}{3} = \boxed{12000}$

b) $27000 = 12000(2) + b$

$27000 = 24000 + b$

$3000 = b$

$\boxed{y = 12000x + 3000}$

$100000 = 12000x + 3000$

$97000 = 12000x$

$\boxed{8.08 \text{ yrs} = x}$

59 (20, 13900) (10, 7500)

a) $\frac{139000-7500}{20-10} = \frac{6400}{10} = \boxed{640}$

$7500 = 640(10) + b$

$7500 = 6400 + b$

$b = 1100$

$\boxed{y = 640x + 1100}$

b) $23500 = 640x + 1100$

$22400 = 640x$

$\boxed{x = 35}$

c) $1000 = 640x + 1100$

$-100 = 640x$

$\boxed{\text{none!}}$

60 (3, 37000) (12, 28000)

a) $\frac{37000-28000}{3-12} = \frac{9000}{-9} = -1000$

$37000 = -1000(3) + b$

$37000 = -3000 + b$

$40000 = b$

$\boxed{y = -1000x + 40000}$

b) $y = -1000(15) + 40000$

$\boxed{= 25000}$

c) $0 = -1000x + 40000$

$-40000 = -1000x$

$\boxed{x = 40}$

(d) $45000 = -1000x + 40000$

$5000 = -1000x$

$-5 = x$

$\boxed{\text{not possible}}$

(61) $(4, .17)\ (7, .33)$

(a) $\frac{.33 - .17}{7 - 4} = \frac{.16}{3} = .053$

$.17 = .053(4) + b$

$.17 = .213 + b$

$-.043 = b$

$\boxed{y = .053x - .043}$

(b) $.5 = .053x - .043$

$.543 = .053x$

$\boxed{x = 10.25 \text{ yrs}}$

(62) $(40, 177)\ (43, 185)$

(a) $\frac{185 - 177}{43 - 40} = \frac{8}{3} = 2.67$

$117 = 2.67(40) + b$

$117 = 106.67 + b$

$b = 70.33$

$\boxed{y = 2.67x - 70.33}$

(b) $y = 2.67(38) + 70.33$

$y \approx \boxed{171.79}$ cm

$y = 2.67(45) + 70.33$

$\boxed{190.48}$ cm

between

(c) $190 = 2.67x + 70.33$

$119.67 = 2.67x$

$x \approx \boxed{44.82 \text{ cm}}$

63 $(.45, .325)\ (.6, .7)$

a $\dfrac{.7-.325}{.6-.45} = \dfrac{.375}{.15} = 2.5$

$.7 = 2.5(.6)+b$
$.7 = 1.5+b$
$-.8 = b$

$\boxed{y = 2.5x - .8}$

b $y = 2.5(.5) - .8$
$= .45$ $\boxed{45\%\ \text{seats}}$

c $.6 = 2.5x - .8$
$1.4 = 2.5x$
$x = .56$ $\boxed{56\%\ \text{votes}}$

d this could be used as a general predictor, but it changes!

64 $(0, 86821)\ (10, 140{,}289)$

a $\dfrac{140{,}289 - 86{,}821}{10-0} = \dfrac{53468}{10} = 5346.8$

$86821 = 5346.8(0) + b$
$86821 = b$

$\boxed{y = 5346.8x + 86821}$

b $y = 5346.8(27) + 86821 = \boxed{231184.6}$

65 $(0, 152000)\ (39, 330{,}870)$

a $\dfrac{330870-152000}{39} = \dfrac{178870}{39} = 4586.41$

$\boxed{y = 4586.41x + 152000}$

b $y = 4586.41(57) + 152000$
$= \boxed{413425.37}$